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Quantum Hall Effect

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The gap equation for the electron self-energy function is considered in the framework of (2 + 1)-dimensional quantum electrodynamics. The filling factor ν for the quantum Hall effect is related to a free parameter λ by considering the development of the gap equation. The latter is shown to be expandable in the power series of m^2/p^2, with p being the modulus of the momentum of a single Hall electron and m its physical mass.

The quantum Hall effect occurs when a two-dimensional system of electrons is subject to a uniform and strong magnetic field (Prange and Gervin, 1990). It is characterized by quantized values of the Hall conductivity, i.e.,

$$\sigma_H = \nu \frac{e^2}{h} \tag{1}$$

where $\nu = 1, 1/3, 2/3, 1/5, 2, \ldots$, the ratio between the number of electrons and the degeneracy of the Landau levels, is interpreted as the filling factor. When the factor ν takes integer values, the Landau levels are completely filled and the integer quantum Hall effect (IQHE) takes place. It is believed that this phenomenon is related to the transport properties of noninteracting electrons (Klitzing, *et al.*, 1980). Furthermore, when the factor ν takes rational values with an odd denominator, the Landau levels are only partially filled: this is the fractional quantum Hall effect (FQHE), which results from the repulsive Coulomb interaction between electrons (Tsui *et al.*, 1982).

The purpose of this paper is to develop an alternative approach, by using a development in powers of m^2/p^2 instead of the method of Acharya and Swamy (1994). We discuss the filling factor characterizing the QHE just by considering this development. To start, let us recall some results derived in

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Acharya and Swamy (1994; hereafter referred to as AS). We make use of the Dyson–Schwinger equation in (2 + 1)-dimensional quantum electrodynamics (in the convention $c = \hbar = 1$):

$$S^{-1} = \not{p} - i\frac{e^2}{(2\pi)^3}\int d^3k\, \gamma^\mu D_{\mu\nu}(p-k)S(k)\gamma^\nu \tag{2}$$

where

$$S(p) = \frac{A(p^2)\not{p} + \Sigma(p^2)}{A^2(p^2)p^2 - \Sigma^2(p^2)} \tag{3}$$

$$D_{\mu\nu}(p) = \frac{1}{p^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right]$$

are the full exact electron propagator when the electrons are interacting and the free photon propagator, respectively.

In AS the authors solved nonperturbatively the integral equation of the self-energy $\Sigma(p^2)$:

$$\Sigma(p^2) = -\frac{2ie^2}{(2\pi)^3}\int d^3k\,\frac{\Sigma(k^2)}{(p-k)^2[k^2 - \Sigma^2(k^2)]} \tag{4}$$

The idea of the approach in AS is to replace the function $\Sigma(p^2)$ of the denominator of the integrand by

$$\Sigma(p^2) \approx m \tag{5}$$

and to transform this function to the Euclidean metric, such that the gap equation (4) becomes

$$\Sigma(p^2) = \frac{2e^2}{(2\pi)^3}\int d^3k\,\frac{\Sigma(-k^2)}{(p-k)^2[k^2 + m^2]} \tag{6}$$

Hence, integrating (6), by use of the Fourier transformation, i.e.,

$$F(r) = \frac{1}{(2\pi)^3}\int d^3pF(p^2)e^{-ip\cdot r} \tag{7}$$

where

$$\Sigma(-p^2) = (p^2 + m^2)\, F(p^2) \tag{8}$$

one finds

$$\Sigma(-p^2) = \frac{m^3}{p^2 + m^2} \tag{9}$$

which defines the so-called full-fledged field theory of the quantized Hall effect. Considering the filling factor

$$\nu = \frac{2}{3\pi}\int_0^{\infty} p^2\,dp\,\frac{3\Sigma(-p^2) + 2p\Sigma'(-p^2)}{[p^2 + m^2]^2} \tag{10}$$

they demonstrated that indeed the theory does account for the FQHE in the case of interacting electrons. Remarkably the relation (10) leads to an approximate solution of the FQHE which is $\nu = 1/3$, but I have the following comments:

1. The presented theory leads to a single, fixed value of ν! This is in sharp contradiction to the fact ν is a continuous parameter which is positive or zero and which can be arbitrarily chosen by the experimentalist.
2. The final result for ν must not be multiplied by two, since in the typical case there is no spin degeneracy (because of the high magnetic field). Hence in the first of the two studied cases the presented theory does not reproduce the IQHE (contrary to what is claimed), but actually would correspond to a FQHE with the fraction 1/2 (Jellal, 1997; Halprin *et al.*, 1993).
3. The results of the calculation lead to negative values of ν. But ν can only be positive or zero.
4. The presented theory leads to a single value for ν (i.e., to a single, quantized plateau). But the QHE consists of a whole series of quantized plateaus.

Having briefly described the method of AS, I turn now to a different approach dealing with the handling of (9). The key point in this approach is to note that in relativistic quantum mechanics, the scalar product of the momentum vector p_μ is

$$p^2 = p^\mu p_\mu = m_0^2 \tag{11}$$

where

$$p_\mu = (m, m\bar{v}) \tag{12}$$

takes a definite sign in the Euclidean region of the (2 + 1)-Minkowski space. This implies that the

$$\frac{m^2}{p^2} < 1 \tag{13}$$

must hold. Indeed, starting from (11) or equivalently

$$m_0^2 = m^2 + m^2 \bar{\nu}^2 \tag{14}$$

one gets the following obvious inequality:

$$\frac{m^2}{m_0^2} = \frac{1}{1 + \bar{\nu}^2} < 1 \tag{15}$$

Then from equation (11) we obtain (13). This last relation is the basis for our further analysis.

Taking in to account (13), we can rewrite the scalar $\Sigma(p^2)$ as series in powers of m^2/p^2 as

$$\Sigma(-p^2) = m \sum_{n=0}^{\infty} (-1)^n \left(\frac{m}{p}\right)^{2n+2} \tag{16}$$

Moreover, setting $x^2 = m^2/p^2$ with $0 \leq x < 1$, we obtain for the filling factor reads as

$$\nu = \frac{2}{3\pi} \int_0^{\lambda} dx \, \frac{\sum_{n=0}^{\infty} (-1)^n x^{2n+2} (5 + 2n)}{(1 + x^2)^2} \tag{17}$$

where $\lambda < 1$ is a free parameter to be specified later.

Since 0 is usually less than one, we can approximate the previous equation by

$$\nu = \frac{2}{3\pi} \int_0^{\lambda} dx \sum_{n=0}^{\infty} (-1)^n \, x^{2n+2} \, (5 + 2n) \tag{18}$$

By integrating this equation, we obtain

$$\nu = \frac{2}{3\pi} \sum_{n=0}^{\infty} (-1)^n \lambda^{2n+3} \, \frac{2n + 5}{2n + 3} \tag{19}$$

which can be written as

$$\nu = \frac{2}{3\pi} \left[\sum_{n=0}^{\infty} (-1)^n \lambda^{2n+3} + 2 \sum_{n=1}^{\infty} (-1)^n \, \frac{\lambda^{2n+1}}{2n + 1} \right] \tag{20}$$

After some elementary manipulations, we find the following λ-dependent function for the filling factor:

$$\nu = \frac{2}{3\pi} \left(\frac{3\lambda^3 + 2\lambda}{1 + \lambda^2} - 2 \, \text{arctg} \, (\lambda) \right) \tag{21}$$

Note that λ is a parameter of the theory which satisfies the condition $\lambda < 1$. But the relevant questions need to be addressed of whether experiment can allow us to fix the free parameter λ, and in which physical situations.

In this paper, by considering a development of the gap equation, we found a relationship between the filling factor ν characterizing the quantum Hall effect and a free parameter λ.

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